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Positive vacuum energy and the N-bound

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Abstract: We argue that the total observable entropy is bounded by the inverse of the cosmological constant. This holds for all space-times with a positive cosmological constant, including cosmologies dominated by ordinary matter, and recollapsing universes. The argument involves intermediate steps which may be of interest in their own right. We note that entropy cannot be observed unless it lies both in the past and in the future of the observer's history. This truncates space-time to a diamond-shaped subset well-suited to the application of the covariant entropy bound. We further require, and derive, a novel Bekenstein-like bound on matter entropy in asymptotically de Sitter spaces. Our main result lends support to the proposal that universes with positive cosmological constant are described by a fundamental theory with only a finite number of degrees of freedom.

Keywords: M-Theory, Black Holes.

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Contents

1. Introduction

1.1 Banks's proposal

Banks [1] has proposed that the cosmological constant should not be viewed as an effective parameter to be derived in a theoretical framework like QFT or string theory. Instead, it is determined as the inverse of the number of degrees of freedom, N, in the fundamental theory.[1] It should thus be considered an input parameter at the most fundamental level of physics.

The proposal can be motivated as follows. In the presence of a positive cosmological constant, Λ, the universe tends to evolve to empty de Sitter space. de Sitter space has a finite entropy $S = 3\pi/\Lambda$, given by the area of the cosmological horizon. Thus the universe is most economically described by a theory with the corresponding

[1] In this paper, N always denotes the number of degrees of freedom; it should not be confused with the size of a gauge group, or the level of supersymmetry. The space-time dimension is taken to be 4 in order to keep equations simple, but generalization to arbitrary dimensions is trivial. Planck units are used throughout.

number of degrees of freedom, $N = 3\pi/\Lambda$. Conversely, a quantum gravity theory with a finite number of degrees of freedom, N, requires for consistency a cosmological constant $\Lambda = 3\pi/N$ to provide a geometric entropy cutoff.

The Λ-N correspondence does not solve the cosmological constant problem except by fiat. It is not clear why the fundamental theory should happen to possess the enormous but finite number of degrees of freedom $N \sim 10^{122}$ that corresponds to the observationally favoured value of the cosmological constant. But the proposal offers a radical, and potentially fruitful, change of perspective.

Its most profound implication is the following: a quantum gravity theory with an infinite number of degrees of freedom, such as M-theory, cannot describe space-times with a positive cosmological constant.[2] This is consistent with the fact that no stable de Sitter vacua are known in M-theory. If the proposal is correct, this gap would not be due to our limited understanding of the theory, but must be ascribed to an obstruction in principle.

The correspondence thus suggests that one should look for a theory with finite N that is self-consistent and *complete*; i.e. it will not do to impose a naive cut-off on an $N = \infty$ theory. If such theories exist for arbitrarily large values of N, one might expect them to limit to M-theory. However, finite-N theories will contain certain qualitative features, such as positive vacuum energy and perhaps supersymmetry breaking,[3] which would be entirely absent in the infinite-N limit and could not have been studied there.

How can the proposal be tested? It asserts that a universe with $\Lambda > 0$ is a system with $N = 3\pi/\Lambda$ degrees of freedom. Unfortunately, this cannot be verified at the semi-classical level, as we have no understanding what the true degrees of freedom are. However, a system with N degrees of freedom certainly cannot have entropy greater than N. Thus, the Λ-N correspondence predicts that a universe with $\Lambda > 0$ cannot have entropy greater than $N = 3\pi/\Lambda$. We call this prediction the *N-bound.* It can be tested.

It is not difficult to see that the N-bound is true for vacuum solutions like de Sitter space (a trivial case). Moreover, one can argue that it is satisfied for all space-times which are asymptotically de Sitter at late times, by the generalized second law of thermodynamics. Indeed, in ref. [1] the Λ-N correspondence was

[2]For a quantum theory, N is defined to be the logarithm of the dimension of Hilbert space; thus, a theory with finite N has a finite-dimensional Hilbert space. Superstring theories, and current non-perturbative proposals for M-theory [2, 3, 4], have an infinite-dimensional Hilbert space. (Indeed, even a single harmonic oscillator has an infinite-dimensional Hilbert space.)

[3]Referece [1] also explores the possibility of a connection between finite N and supersymmetry (SUSY) breaking, noting that no stable SUSY-violating vacuum states have been firmly identified in M-theory (see, however, refs. [5, 6, 7]). One may therefore speculate that SUSY breaking can only occur in theories with finite N, and that both the SUSY and the vacuum energy scales arise from finite N. One then faces the challenge of explaining why the SUSY scale is much larger than N^{-1}. We shall not pursue the connection with supersymmetry in the present paper.

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conjectured to apply only within this class of space-times. This includes, for example, the $\Lambda > 0$ flat Friedman-Robertson-Walker (FRW) solution which appears to describe our universe: it starts out with a big bang and is initially radiation — or matter-dominated; then the matter is diluted by the cosmological expansion; and at some time (as it happens, roughly now) the vacuum energy — which is not diluted — starts to dominate and leads the universe to evolve towards empty de Sitter space in the far future.

However, solutions with $\Lambda > 0$ need not necessarily become de Sitter at late times. Consider, for example, the time reversal of the cosmological solution just described: it starts out as empty de Sitter; then more and more matter condenses, which eventually causes the space-time to collapse in a big crunch. This illustrates, in particular, that one can never be sure to have reached the safety of asymptotic de Sitter space; there is always the possibility of a huge collapsing shell of matter that cannot be seen yet but will cause an apocalypse in the future. Another example is a $\Lambda > 0$ closed FRW universe. Given a sufficiently large matter density, the cosmological constant will not be strong enough to prevent recollapse. Indeed, Λ might be a negligible contribution to the total energy density at all times.

These $\Lambda > 0$ solutions are perfectly valid from the perspective of semi-classical gravity. Many of them are physically quite reasonable, and we would find it unconvincing to exclude them *a priori*. In some cases, a small perturbation can make all the difference between collapse and expansion to asymptotically de Sitter space. These arguments lead us to advocate a stronger version of Banks's proposal. We conjecture that the Λ-N correspondence holds for all $\Lambda > 0$ universes, including those that do not evolve to de Sitter in the future. But if de Sitter is not the "final state", the second law will be of no help, and it is no longer obvious that the N-bound holds. The N-bound is thus a non-trivial prediction of the Λ-N correspondence.

Indeed, at first sight, some solutions may appear to have entropy greater than N, in contradiction with the correspondence. Nevertheless, it will be argued in this paper that the N-bound is valid for all universes with $\Lambda > 0$. This statement is far from obvious, and its proof will be seen to require a number of non-trivial intermediate results. Therefore, our conclusion may be viewed as evidence in favour of the proposed correspondence.

1.2 Outline

Our goal is to prove the following conjecture:

N-bound. *In any universe with a positive cosmological constant Λ (as well as arbitrary additional matter that may well dominate at all times) the observable entropy S is bounded by*

$$N = \frac{3\pi}{\Lambda}\,. \tag{1.1}$$

Here S includes both matter and horizon entropy, but excludes entropy that cannot be observed in a causal experiment. Note that N is the Bekenstein-Hawking entropy of empty de Sitter space. The bound becomes trivial in the limit of vanishing cosmological constant. As we have argued above, an independent proof of the N-bound provides strong support to the proposed Λ-N correspondence [1]; hence, the correspondence will not be used anywhere in the paper.

In section 2 we ask what constitutes observable entropy. We argue that one should restrict attention to the *causal diamond* of an observer: the space-time region that can be both influenced and seen by an observer. Thus, the observable entropy lies in a region bounded by the past and future light cones from the endpoints of the observer's world line.

The covariant entropy bound [8]–[11], reviewed in section 3, can be applied to the cones bounding the observable region. This turns out to imply only $S \leq 2N$, however, which does not quite suffice. In section 4, we derive a novel bound on the entropy of matter systems in de Sitter space, the "D-bound", which can be tighter than the covariant bound. In section 5 we argue that the two bounds can be combined to imply the N-bound. The results are discussed in section 6.

Banks's discussion [1] of the consistency of his proposal involved many of the considerations that enter our derivation of the N-bound. The arguments presented here are strongly influenced by Susskind's emphasis on the operational meaning of physical quantities. The covariant entropy bound [8], which plays a central role in the present work, generalizes a proposal by Fischler and Susskind [12] and is thought to have its origin in the holographic principle, first formulated by 't Hooft [13] and Susskind [14]. The application of the covariant entropy bound to the past light-cone of an observer was proposed by Banks in ref. [15]. The D-bound is related to Bekenstein's bound on the entropy of finite systems in flat space [16]. Its derivation adapts the original arguments of Geroch and Bekenstein, and extends to cosmologically large systems Schiffer's use of the cosmological horizon to obtain Bekenstein's bound [17]. Bekenstein's generalized second law of thermodynamics [18, 19, 20] underlies most of the work in this paper. The semi-classical description of asymptotically de Sitter space-times was laid out by the work of Gibbons and Hawking [21]; see also ref. [22]. Other recent work exploring connections between the holographic principle and the cosmological constant includes refs. [23, 24, 25].

2. Causal diamonds

We first address the question of which entropy (or information) is actually accessible to a given observer. We will argue that certain space-time regions can be eliminated from consideration, and that the N-bound need only hold for the remaining region,

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the "causal diamond" associated with an observer. It will also be shown that these restrictions are necessary, in the sense that the inclusion of unobservable entropy would easily allow the violation of the bound.

Implicit in this approach is the principle, long advocated by Susskind, that a fundamental theory need only answer questions that are operationally meaningful. For example, it need not (and, from an aesthetic standpoint, should not) simultanously describe the experiments made by two separate observers who, for reasons of causal structure, will never be able to compare results. Of course, it must be able to describe each experiment separately. This principle has previously been used to resolve certain apparent paradoxes in the evaporation of black holes [26, 27, 28].

We will consider an experiment that begins at point p and ends at a later point q on the observer's world line. It will be seen that causality limits the space-time region whose entropy can play a role in the experiment. It may be sufficient to consider only "the longest experiment possible", i.e. the limit in which p is taken to be in the far past, and q in the far future, on the world line. However, it will be simpler and more instructive to carry out the discussion for arbitrary p and q. As experiments often have finite duration, this is the most general case; and all results will continue to hold in the limit of early p and late q.

2.1 The past light-cone

There are two independent restrictions. The first is:

(R1) *Consider only the observer's causal past, $J^-(q)$. Ignore everything else.*

This is a sensible restriction. At the point q, the endpoint of the experiment, the observer can only have received signals from the past of q. The rest of space-time has not yet been seen. For the purposes of the experiment in question, its entropy is operationally meaningless and can be ignored.

For the later application of entropy bounds, note that the observer's past is bounded by the past light-cone from the point q, and that all matter within the observer's past must pass through this cone.[4] Thus, if one wishes to bound the observable entropy, it will be sufficient to bound the entropy on the past light-cone of the endpoint, q.

[4] This is intuitively obvious but can be made precise as follows. The causal past of q, $J^-(q)$, is defined as the set of points that can be reached from q via a smooth curve that is everywhere past-directed timelike or null. Define the past light-cone, $L^-(q)$, as the hypersurface generated by the past-directed null geodesics that start at q and are terminated if and only if they run into a point conjugate to q (a "caustic"). Assuming global hyperbolicity one can show [29] that the boundary of the past of q, $\dot{J}^-(q)$, is a portion of $L^-(q)$. For any point $r \in J^-(q)$, we claim that all future inextendible causal curves through r must intersect $L^-(q)$. In fact, the stronger statement holds that they must intersect $\dot{J}^-(q)$; in the notation of Wald [29], $D^-[\dot{J}^-(q)] = J^-(q)$. This follows from the compactness of $J^+(r) \cap J^-(q)$ Wald [29, (Theorem 8.3.10) and Lemma 8.2.1].

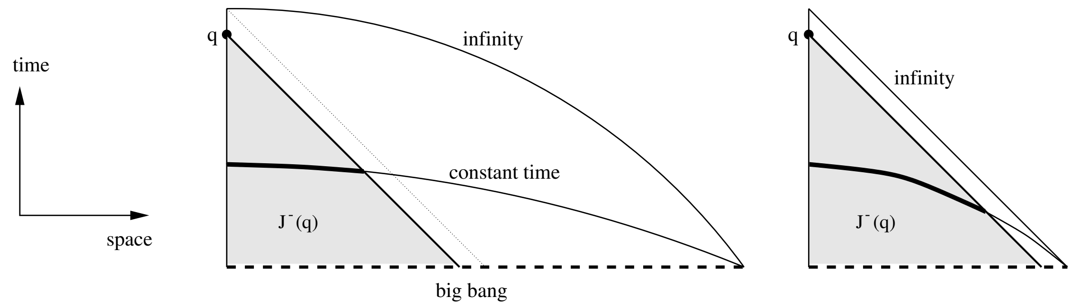

Figure 1: Flat FRW universe with $\Lambda > 0$ (left). The entropy on any constant-time slice is infinite, but only a finite portion (heavy line) can be seen by the observer at q. Because of the future de Sitter horizon (dotted line), this portion will not diverge. Right: flat FRW universe with $\Lambda = 0$. The entropy within the observer's past light-cone diverges at late times.

The restriction R1 is necessary for the N-bound. Consider a $\Lambda > 0$ flat FRW universe starting with a big bang — possibly a good approximation to the universe we inhabit. The entropy density on any homogeneous spacelike slice is constant; thus, the total entropy on the slice is formally infinite, in apparent violation of the N-bound. The restriction R1 resolves this problem. Because the cosmological constant dominates at late times, any observer has a future event horizon (figure 1). The entropy in its interior is finite. Because the event horizon contains the observer's past for any endpoint q, the observed entropy is also finite. (We do not show quantitatively that it satisfies the N-bound as this will follow from the general arguments given in section 5.)

In this example, space-time is asymptotically de Sitter in the future, with entropy N. Thus, R1 is not only sensible and necessary for the N-bound, but indeed necessary for the validity of the generalized second law of thermodynamics.

It is instructive to contrast the above example with the case of a $\Lambda = 0$ flat FRW universe (figure 1). The latter has a different infinity structure. Arbitrarily large portions of any flat hypersurface lie within the past light-cone at sufficiently late times. Even with restriction R1, the observed entropy is unbounded. Of course, this is not a problem, because $N = \infty$ in this case.

2.2 The future light-cone

The second restriction is:

(R2) *Consider only the observer's causal future, $J^+(p)$. Ignore everything else.*

Note that the observer's future is bounded by the future light-cone of the point p, and that all matter within the observer's future must have entered through this cone.[5]

[5]This follows by exchanging "past" and "future", $-$ and $+$, and q and p, in the previous footnote.

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This restriction may seem less obvious than the previous one. But it is just as sensible. It is not enough for entropy, or information, to lie in the observer's past. To be observed, it actually has to get to the observer, or at least to a region that can be probed by the observer. But an experiment that commences at p can only probe what is in the causal future of p.

Put differently, all information that reaches the observer, or at least is accessible to the observer, must have passed through the future light cone of p. For the purpose of describing the experiment in question, one can ignore the space-time region outside the cone; instead, one may think of the initial conditions as residing on the cone. Entropy that fails to enter through the cone is operationally meaningless: though it may well be present in the observer's causal past, an experiment that starts at p will not know about it, because it cannot probe the region where such entropy resides.

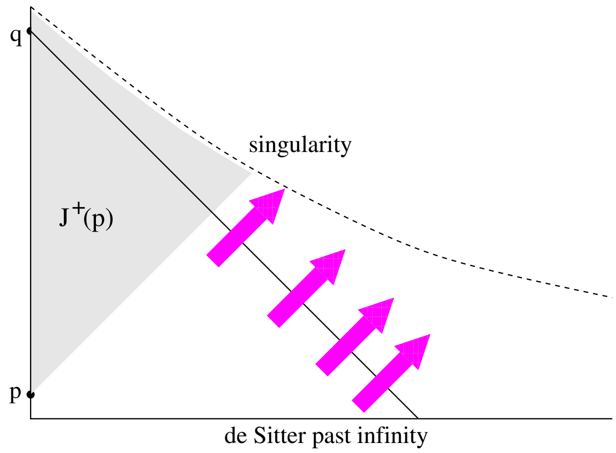

Figure 2: A collapsing universe approaching de Sitter in the past. The past light-cone of q may contain infinite entropy (arrows), but only a finite amount will enter the future of p (shaded region).

How is this consistent with cosmological observations of distant galaxies? By measuring the cosmic microwave background radiation, are we not collecting information about the early universe? These regions are indeed outside the future of our entire world-line, let alone the future of the point when the experiment began. However, all the information we gathered was in photons that interacted with some local apparatus. They had to enter through the future cone to get here. So the entropy we actually observe is quite local. It is certainly insightful to interpret this information in terms of models that involve inaccessible regions. For example, one might say that the early universe contained certain density perturbations. But the information used to obtain this conclusion is here, now. Thus, it is subject to entropy bounds associated with a much smaller region than the one it is interpreted to be an imprint of.

Without the additional restriction R2, the N-bound would fail. Figure 2 shows a $\Lambda > 0$ space-time in which the observer's causal past contains an arbitrarily large entropy. Consider a universe that approaches empty de Sitter space asymptotically in the past. The geometry will resemble the lower half of the de Sitter hyperboloid at early times (see appendix A). It contains exponentially large three-spheres, on which one can place dilute matter with arbitrary entropy. If the total entropy exceeds N, the universe will necessarily be dominated by this matter at a later time. It will collapse in a big crunch, and there will be no future de Sitter region. One can arrange for the energy and entropy density to be constant on the observer's past light-cone (by giving it an increasing profile on the early S^3, in the radial direction away from the observer's world line). The past light-cone keeps going forever, and so the total entropy on it will be infinite. — Note that the area of surfaces on the past light-cone diverges, so this example does not contradict the covariant entropy bound discussed in section 3.

2.3 The causal diamond

Recall that p and q are two points on an observer's world line, with q later than p. One can think of p as the beginning and q as the end of some experiment. The restrictions R1 and R2 define the space-time region that can come into play in such an experiment. According to R2, one can ignore what is outside the causal future of p, and R1 states that regions outside the causal past of q are operationally meaningless as well. Combining both conditions, one can restrict to the points which are both in the future of p and in the past of q. This set,

$$C(p, q) = J^+(p) \cap J^-(q) \,, \tag{2.1}$$

will be called the *causal diamond* associated with an experiment beginning at p and ending at q. Thus, one obtains the condition

(R1+R2) *Consider only the entropy in causal diamonds, i.e. in regions of the form $C(p, q)$.*

(The notion of an observer's world line was a crutch that can be dropped now. If q is in the future of p, there will be world lines connecting them; if not, then $C(p, q)$ will be empty or degenerate.)

Of a fundamental theory, one may demand that it describe any experiment, but no more than that. Hence, it should describe the physics in any causal diamond, that is, in any region of the form $C(p, q)$ for some pair of points (p, q), but *only one causal diamond at a time*. One should not demand that the theory simultaneously describe two separate causal diamonds, unless they are both contained in a single larger causal diamond.

For example, the theory should be able to describe an experiment inside a black hole, as well as an experiment outside a black hole. But it should not describe correlation functions between a point inside and a point outside a black hole if those points do not lie in any causal diamond. This example is just a reformulation of some of the arguments that established the concept of "black hole complementarity" [26, 27, 28]. (In this case only the restriction R1 really matters, since R2 can easily be satisfied.) An analogous argument can be made for pairs of points near a big bang singularity. If they are sufficiently far, they cannot lie in a single causal diamond. Then no experiment can be set up that will involve both points. (In this case, R2 is the crucial restriction.)

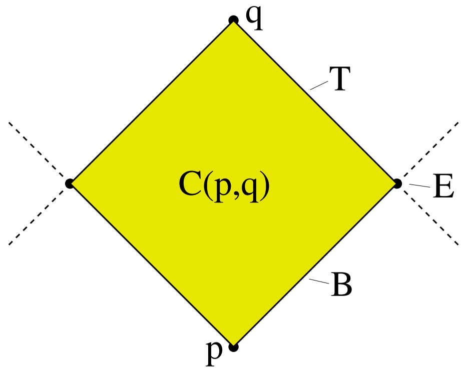

Figure 3: A causal diamond, with top cone T, bottom cone B, and edge E.

In space-times that are asymptotically de Sitter in the past and future, any causal diamond lies within both the past and future event horizon. (Both R1 and R2 are used here.) The exponentially large regions beyond those horizons are operationally meaningless. This result has long been advocated by Susskind.

A causal diamond is bounded by a *top cone* (a portion of the past light-cone of q), and a *bottom cone* (a portion of the future light-cone of p); see figure 3. The cones usually, though not necessarily, intersect at a two-dimensional spatial surface, the *edge* of the causal diamond. In any case, the entropy in the causal diamond must pass through the top cone (and all matter must have entered through the bottom cone).[6] It will be seen below that the nature of the boundaries allows for a straightforward application of the covariant entropy bound. For this reason, the entropy within a causal diamond is under good theoretical control.

3. The covariant entropy bound

The covariant entropy bound [8] bounds the entropy on certain null hypersurfaces or "light-sheets". It was developed in order to formulate the holographic principle [13, 14] for general space-times [9], and can be viewed as a generalization of the approach of Fischler and Susskind [12]. The use of null hypersurfaces to relate entropy and area was originally suggested by Susskind [14]. Several concepts crucial to a general formulation were first recognized by Corley and Jacobson [30].

The bound is conjectured to hold for any spacial surface in any space-time with reasonable energy conditions. It will be useful here because it applies even to regions, such as recollapsing universes or black hole interiors, where the second law is of no help. The conjecture has passed a number of non-trivial tests [8]. It has been proven in space-time regions where a fluid approximation to entropy can be made with plausible relations between entropy and energy density [11].

[6] This follows from the previous two footnotes, with $\dot{C}(p,q) = [J^+(p) \cap \dot{J}^-(q)] \cup [\dot{J}^+(p) \cap J^-(q)]$. The first term in square brackets is the top cone, $T(p,q)$; the second is the bottom cone, $B(p,q)$; their intersection is the edge, $E(p,q)$. Clearly, $T(p,q) \subset \dot{J}^-(q) \subset L^-(q)$, and similarly, $B(p,q) \subset L^+(p)$.

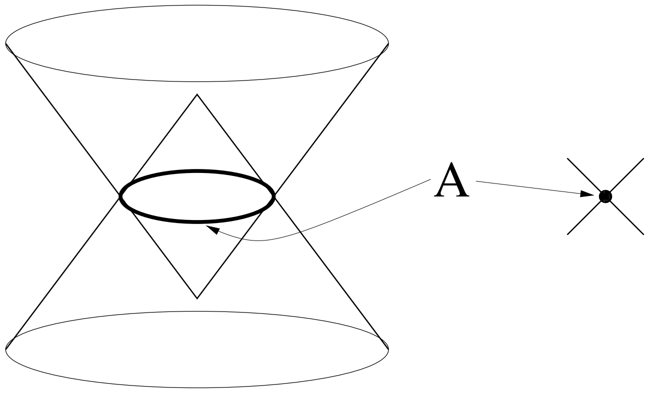

Figure 4: The four light-like hypersurfaces orthogonal to a spatial surface (in this example, two cones going in and two "skirts" going out). In a Penrose diagram the four null directions are indicated by an "X" (right).

Consider some two-dimensional spatial surface of area A. (We will mostly be interested in closed surfaces, but this is not a necessary restriction.) Any surface has four orthogonal light-like directions. Namely, the surface has two sides, and on each side there is a family of orthogonal light-rays arriving from the past (past-directed light-rays), and a family of future-directed light-rays. In figure 4 this is illustrated for the example of a spherical surface. In a Penrose diagram, where light travels at 45 degrees, the four orthogonal light-like directions are indicated by the legs of an "X" centered on the point that represents the sphere.

The orthogonal light-rays generate four 2+1-dimensional null hypersurfaces. On some of them, the *light-sheets* of the surface A, the cross-sectional area spanned by the light-rays will be decreasing or constant in the direction away from the original surface. (In the example in figure 4, the two cones.) The entropy on any light-sheet is less than $A/4$:

$$S(\text{light-sheet of } A) \leq \frac{1}{4} A \,. \qquad (3.1)$$

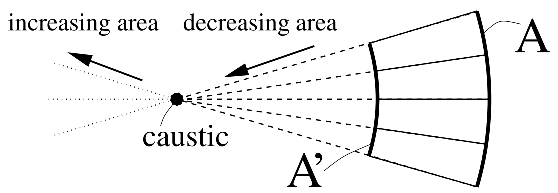

Figure 5: A light-sheet is a null hypersurface with shrinking cross-sectional area. It terminates at caustics (if not earlier).

Any surface has at least two light-sheets, since two of the four families of light-rays are just continuations of the opposite two. E.g., if the area is increasing in the future direction to one side, it will necessarily decrease in the past direction to the other side. If it is constant in some direction, both opposing legs will be allowed.

The requirement of decreasing cross-sectional area is a local condition and it must hold everywhere on the light-sheet. This means that the light-sheet must be

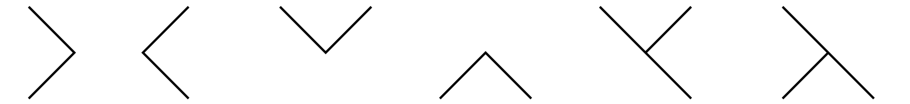

Figure 6: Wedge symbols for different types of surfaces. A leg is drawn for each direction in which light-rays are non expanding.

terminated at or before one reaches a caustic, i.e. before neighbouring light-rays intersect (figure 5). In figure 4, the tips of the cones are caustics, and the light-sheets end there. The focussing theorem guarantees that contracting light-rays will not become expanding without going through a caustic.[7] If one chooses to terminate the light-sheet before each light-ray reaches a caustic, the end-points will span a non-zero area A'. Then the covariant bound can be strengthened [11]:

$$S \leq \frac{1}{4}\left(A - A'\right). \tag{3.2}$$

The light-sheet directions associated with a surface can be indicated, in a causal diagram, by the corresponding legs of the "X" (figure 6). The two allowed directions form a wedge. One may classify closed surfaces as follows. For *normal* surfaces, both legs of the wedge point to one side, which is called the *inside* by definition. If both light-sheets are future-directed, the surface is *trapped*; if the area is contracting in both past directions, it is called *anti-trapped.* Marginal cases arise for surfaces on the interface between a normal and a trapped or anti-trapped region. Then the expansion vanishes along at least one opposing pair of legs, and three or four legs must be drawn. The covariant entropy bound is particularly powerful when applied to such surfaces, and we will focus on them in section 5.

4. The D-bound on matter entropy in de Sitter space

By studying the second law of thermodynamics in asymptotically flat space, Bekenstein found that the total entropy is given by the sum of ordinary matter entropy, $S_{\rm m}$, plus the semiclassical Bekenstein-Hawking entropy, $S_{\rm h} = \frac{1}{4}A_{\rm h}$, associated with the horizons of black holes [18, 19, 20, 33]. Similarly, in asymptotically de Sitter

[7] The null convergence condition [31] is assumed to hold: $T_{ab}k^a k^b \geq 0$ for all null vectors k^a. — It has been suggested [32] that a light-sheet be terminated also at points where non-neighbouring light-rays intersect. As this can only make the light-sheet smaller, it gives a weaker bound, but the smaller light-sheet may be easier to compute practically. The light-sheets in section 5 below are of this simple type, because they are a portion of the boundary of the causal past of a point.

space, the cosmological horizon contributes with

$$S_{\mathrm{c}} = \frac{1}{4} A_{\mathrm{c}} \tag{4.1}$$

to the total entropy [21].

Empty de Sitter space has a cosmological horizon of area

$$A_0 = \frac{12\pi}{\Lambda} = 4N \tag{4.2}$$

(see appendix). Therefore, empty de Sitter space has horizon entropy $S_{\mathrm{c}} = N$. One might think that even a tiny amount of matter entropy would already increase the total entropy, $S_{\mathrm{c}} + S_{\mathrm{m}}$, above N. However, the cosmological horizon surrounding a matter system in asymptotically de Sitter space is smaller than A_0: the more matter, the smaller the cosmological horizon. Thus it is possible that the total entropy remains bounded by N. (It is instructive to verify this explicitly for the simple case of Schwarzschild-de Sitter black holes; see also [34])

It will now be shown that the N-bound is in fact implied by the second law *if* space-time contains an asymptotically de Sitter region in the future. This will allow us, by subtracting the horizon entropy from N, to derive a bound on the matter entropy in de Sitter space. Despite the restrictive assumption of an asymptotic de Sitter region, this bound will be useful to our purpose; we will argue later that it may also be applied to certain portions of more general space-times. Thus it will join the covariant bound, and the concept of causal diamonds, as a third ingredient in the argument constructed in section 5 to show that the N-bound is valid for all $\Lambda > 0$ space-times.

Consider the following process. The initial configuration is a matter system in asymptotically de Sitter space. The matter system may contain black holes, whose entropy is taken to be included in the matter entropy, S_{m}. The system is surrounded by a cosmological horizon of area A_{c}. The final state is empty de Sitter space. The transition is achieved by taking the observer to move into the asymptotic region. (To the observer, the matter system appears to fall into the cosmological horizon.) In this process, the matter entropy S_{m} is lost, while the entropy of the cosmological horizon increases by an amount

$$\Delta S_{\mathrm{c}} = \frac{1}{4} (A_0 - A_{\mathrm{c}}) \, . \tag{4.3}$$

The generalized second law of thermodynamics [18, 19, 20] implies that the total entropy must not decrease:

$$\Delta S_{\mathrm{c}} \geq S_{\mathrm{m}} \, . \tag{4.4}$$

With $A_0 = 4N$, one obtains a bound on the matter entropy:

$$S_{\mathrm{m}} \leq N - \frac{1}{4} A_{\mathrm{c}} \, . \tag{4.5}$$

To distinguish this bound from the covariant entropy bound and the N-bound, it will be called the *D-bound* ("D" as in Difference between N and the horizon entropy).

The D-bound is less general than the covariant bound of section 3, because it only applies to matter systems within a de Sitter horizon. For a dilute system, one has $A_c \approx A_0$, and therefore, $N - \frac{1}{4}A_c \ll \frac{1}{4}A_c$. So the D-bound can be tighter than the covariant bound applied to a surface enclosing the system. In the next section it will be seen that causal diamonds can contain portions to which the D-bound applies.

5. The N-bound

In section 1 the N-bound was presented as a conjecture: the observable entropy in any $\Lambda > 0$ universe cannot exceed $N = 3\pi/\Lambda$. In section 2 it was shown that only the entropy within space-time regions of a particular form, causal diamonds, is observable. Hence, to prove the N-bound, it suffices to show that the entropy of an arbitrary causal diamond does not exceed N. By applying the covariant entropy bound (section 3) and the D-bound (section 4), we will now give a proof for spherically symmetric causal diamonds. Spherical symmetry allows us to keep the discussion fairly non technical and focus on the key idea, the interplay between the D-bound and the covariant bound. We expect that the assumption of spherical symmetry can be eliminated in a more refined treatment; this will be discussed briefly at the end of the section.

Consider an experiment beginning at a point p and ending at q, in a universe with $\Lambda > 0$. We must show that the matter entropy, S_m, within the causal diamond, $C(p, q)$, plus the Bekenstein-Hawking entropy of any black hole or cosmological horizons identified by the experiment, will not exceed N. To limit S_m, it suffices to consider the matter entropy passing through the top cone bounding the diamond, by the second law. It will be seen that the horizon entropy is bounded by the area of the diamond's edge.

Neither the top nor the bottom cone contain any caustics, except at endpoints, because each is a portion of the boundary of the future or past of a point. Then, by the focussing theorem, each cone has exactly one maximal cross-sectional area. The maximum may be local, or it may lie on the intersection of the two cones, the edge, where they terminate. Depending on the location of the maxima, we distinguish three cases.

Case 1: no local maximum on either cone. Then the maximum area of each cone lies on the edge. Thus, the edge will be a normal surface, with the observer on the inside (figure 7). This case applies, for example, to regions within the horizon in an asymptotically de Sitter universe, and to sufficiently small causal diamonds in arbitrary spacetimes.

Consider a space-like hypersurface containing the edge. One can consistently assume, for the sake of argument, that the exterior of the edge is a vacuum solution. With the assumption of spherical symmetry, Birkhoff's theorem implies that the exterior will be a portion of a Schwarzschild-de Sitter (or a Reissner-Nordström-de Sitter) solution. The space-time thus constructed will be called the *auxiliary space-time*. It is asymptotically de Sitter in the future and past; hence, it invites the application of the D-bound.

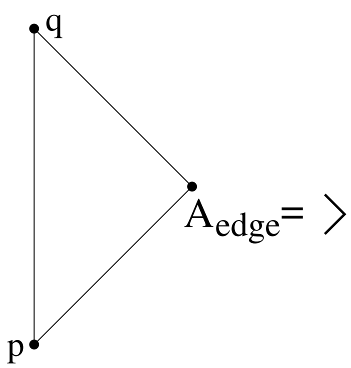

Figure 7: Causal diamond in case 1 (to be rotated about the p-q axis). The edge is normal. The D-bound applies to the top cone.

The causal diamond lies within the cosmological horizon of the auxiliary space-time, because the edge is normal and the cosmological horizon is the outermost normal surface. With spherical symmetry it follows that $\hat{A}_{\rm c} \geq A_{\rm edge}$. The hat indicates that the cosmological horizon is a surface in the auxiliary space-time. Because the auxiliary space-time is asymptotically de Sitter, the D-bound can be applied to the interior of the cosmological horizon, yielding

$$S_{\rm m} \leq N - \frac{1}{4}\hat{A}_{\rm c} \leq N - \frac{1}{4}A_{\rm edge}\,. \tag{5.1}$$

Recall from section 4 that the entropy of black holes is already subsumed in $S_{\rm m}$,[8] but not the entropy of the cosmological horizon (supposing that one exists in the actual space-time under consideration). However, the observer cannot assign more cosmological horizon entropy than a quarter of the area of the outermost surface that has been probed:

$$S_{\rm c} \leq \frac{1}{4}A_{\rm edge}\,. \tag{5.2}$$

The two inequalities imply the N-bound,

$$S = S_{\rm m} + S_{\rm c} \leq N\,. \tag{5.3}$$

Case 2: local maximum on the top cone. Now assume that the top cone contains a locally maximal area $A_{\rm max}$, an *apparent horizon*. No assumption is made about the bottom cone. Cases of this type include large expanding or collapsing cosmological regions. They correspond to highly dynamical situations without a quasi-static cosmological horizon, so $S_{\rm c} = 0$. Then we need to show only that the matter and black hole entropy on the top cone does not exceed N.

[8]This may play a role when spherical symmetry is abandoned. In the presence of black holes, the edge can contain additional disconnected components, namely spherical surfaces surrounding the black hole horizons, within the cosmological horizon.

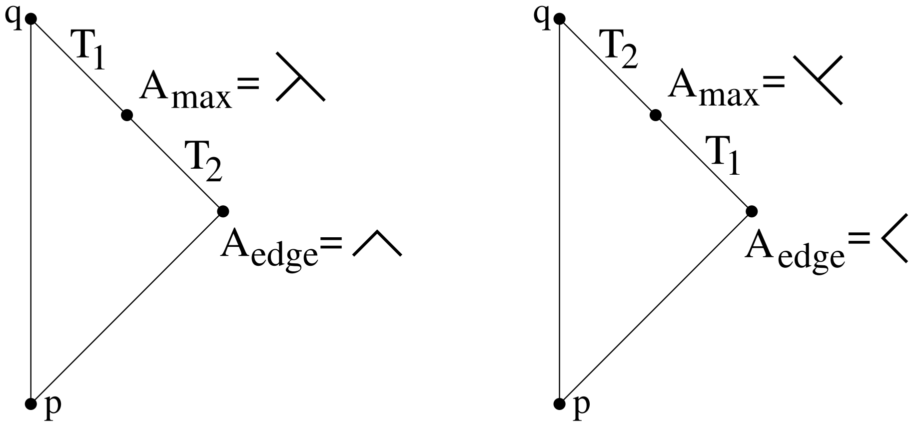

Figure 8: Case 2. One side of the maximal area must be normal. We apply the D-bound to this side, T_1, and the covariant bound to the other side, T_2. Both possibilities are shown.

The maximal area divides the top cone into two parts. We will show that the D-bound can be applied to one part and the covariant bound to the other. Recall the wedge formalism summarized in figure 6. The wedge of the surface $A_{\max}$ is constructed by drawing a leg for each light-like direction with decreasing cross-sectional area. Because $A_{\max}$ is the largest surface on the top cone, the area obviously decreases in the two null directions that generate the cone. Of the other two null directions orthogonal to $A_{\max}$, at least one must have decreasing area, because they oppose each other. Hence, the wedge associated with $A_{\max}$ has at least three legs. Necessarily, two of them will be pointing to the same spatial side (figure 8). Therefore, $A_{\max}$ is a marginally normal surface. The side with two legs is, in the wedge sense, the inside of $A_{\max}$. The corresponding portion of the top cone will be called T_1. Note that T_1 need not be the portion that includes the tip; it may be on the "far side" of $A_{\max}$ (figure 8, right).

Consider the inside portion, T_1, in isolation. To this hypersurface one can apply the D-bound, using an argument similar to that of case 1. One can take T_1 to be embedded in an otherwise vacuous auxiliary space-time. Because T_1 is the interior of a normal surface, in the auxiliary space-time it will be surrounded by a cosmological horizon. The area of the cosmological horizon will be no less than $A_{\max}$. Because the auxiliary space-time is asymptotically de Sitter, the D-bound applies to the interior of the cosmological horizon. Hence, the entropy on T_1 satisfies

$$S_1 \leq N - \frac{1}{4}\hat{A}_{\rm c} \leq N - \frac{1}{4}A_{\max}\,. \tag{5.4}$$

The other part, T_2, of the top cone, is a light-sheet of the surface $A_{\max}$. The covariant entropy bound yields

$$S_2 \leq \frac{1}{4}A_{\max}\,. \tag{5.5}$$

It follows that the entropy on the top cone is bounded by N:

$$S = S_1 + S_2 \leq N\,. \tag{5.6}$$

In this result, S already includes black hole entropy. A horizon is probed by an experiment only if the edge of the causal diamond contains a portion in the vicinity of the horizon. The edge lies on the far side of the top cone. If this is T_1, the side to which the D-bound applies, then the black hole entropy is already subsumed in S_1, as discussed in section 4. If the far side is T_2, let us split S_2 into black hole horizon entropy, $S_{\rm h}$, and ordinary matter entropy, $S_{\rm m}$:

$$S_2 \equiv S_{\rm h} + S_{\rm m}\,. \tag{5.7}$$

If $A_{\rm edge} > 0$, the covariant bound on T_2 can be strengthened [11]:

$$S_{\rm m} \leq \frac{1}{4}\left(A_{\rm max} - A_{\rm edge}\right). \tag{5.8}$$

The horizon cannot be larger than the area of the edge:

$$S_{\rm h} \leq \frac{1}{4} A_{\rm edge}\,. \tag{5.9}$$

So eq. (5.5) holds, and S in eq. (5.6) is indeed the total observable entropy.

Case 3: local maximum on the bottom cone but not on the top cone. Finally, consider the case where the top cone has no local maximum, but the bottom cone does (figure 9). Examples include large regions in collapsing universes, or black hole interiors. The edge of the cone is a trapped surface in this case. This implies a dynamical situation without Bekenstein-Hawking entropy. It will suffice to show that the matter entropy on the top cone does not exceed N.

In the absence of a local maximum, the largest surface on the top cone is the edge. The entire top cone is a light-sheet of the edge. By the covariant entropy bound,

$$S \leq \frac{1}{4} A_{\rm edge}\,. \tag{5.10}$$

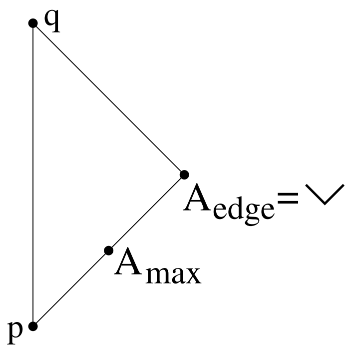

Figure 9: Case 3. The edge is trapped. We apply only the covariant entropy bound.

By the arguments used in case 2, the maximal area on the bottom cone, $A_{\rm max}$, is a normal surface. Hence, it can be embedded in an asymptotically de Sitter auxiliary space time, where it is surrounded by a cosmological horizon of area $\hat{A}_{\rm c}$. By the second law, $\hat{A}_c$ cannot exceed the horizon area of empty de Sitter space, A_0. Moreover, by construction, $A_{\rm max}$ is larger than the edge. In summary, one finds

$$A_{\rm edge} \leq A_{\rm max} \leq \hat{A}_{\rm c} \leq A_0 = 4N\,. \tag{5.11}$$

Therefore the N-bound is satisfied:

$$S \leq N \,. \tag{5.12}$$

In all three cases, we have used an auxiliary construction by which (portions of) the causal diamond were embedded in an asymptotically de Sitter auxiliary space-time. This method is rigorous only for spherically symmetric situations. The assumption was used in applying Birkhoff's theorem to establish the auxiliary space-time, and in taking the cosmological horizon as an upper bound on the area of normal surfaces on the light-cone. Spherical symmetry has also simplified the case distinction, since it implies that the maximum is either local or entirely on the edge; in general, the maximal area on the top cone may have locally maximal components as well as portions that lie on the edge.

Our assumption of spherical symmetry notwithstanding, we expect that the above arguments represent the core of a general proof. Causal diamonds, light-sheets, and the entropy bounds are all defined without reference to spherical symmetry. The task of combining them to derive the N-bound in the non-spherical case is left to future work.

6. Outlook

Non-perturbative definitions of quantum gravity have been given for certain space-times that are asymptotically flat or AdS [2, 4]. No such description has been found for space-times with a positive cosmological constant. As no de Sitter solutions of M-theory are known, one does not even have a microscopic framework. Banks [1] has opened a new perspective on this problem by suggesting that an asymptotically de Sitter universe is described by a microscopic theory with finite-dimensional Hilbert space. Quantitatively, the Λ-N correspondence relates the cosmological constant of a stable vacuum, Λ, to a theory with $N = 3\pi/\Lambda$ degrees of freedom (i.e. with a Hilbert space of dimension e^N).

If this is correct, M-theory (as it is currently understood) will arise only in the limit of vanishing Λ and infinite N. The cosmological constant problem becomes a problem of understanding the particular dimension of Hilbert space chosen for the theory.

In section 1.1, considerations of consistency with semi-classical gravity led us to propose the stronger conjecture that the Λ-N correspondence applies to all universes with $\Lambda > 0$, whether they are de Sitter in the future or not. This conjecture makes the non trivial, testable prediction that the observable entropy in all such universes is bounded by N. We then argued that this statement, the "N-bound", is correct. This required the combination of the covariant entropy bound with two intermediate results derived in sections 2 and 4: the D-bound, and the restriction to causal diamonds. It is hard to see what, other than the Λ-N correspondence, would offer a compelling explanation why such disparate elements appear to join seamlessly to imply a simple and general result.

The D-bound has a number of properties that merit further investigation. In particular, one can show that it is closely related to Bekenstein's bound [16]. Bekenstein's bound, valid for systems in flat space, can be written as $S_{\rm m} \leq \pi r_{\rm g} R$, where $r_{\rm g} = 2m$ is the "gravitational radius" of the system and R is the circumscribing radius. For dilute, spherically symmetric systems in de Sitter space, the D-bound takes precisely this form as well, despite the significant deviation from flat space. A full discussion is given elsewhere [35].

The restriction to causal diamonds arose in section 2 from the requirement to include only operationally meaningful parts of a space-time in a microscopic description. This principle is independent of the present context of positive Λ, and one may expect that causal diamonds will be of wider use. Banks [15] has sketched a framework for the combination of quantum mechanics and cosmology, in which the variable size of the quantum Hilbert space is related to the maximal area of the observer's past light-cone. The arguments of section 2.2 suggest a possible modification of this approach that may lead to a more time-symmetric treatment based on the Hilbert space of causal diamonds.

In de Sitter space, an observer will be immersed in quantum radiation coming from the cosmological horizon. At the semi-classical level, this radiation is thermal [21]. One would expect that the radiation will occasionally contain large fluctuations that appear to an observer as classical objects. Taking a global view of the de Sitter space, one would say that quantum fluctuations originate behind the future horizon, while classical objects enter through the past horizon. When one restricts to causal diamonds, however, both of these outside regions are eliminated. Then it is no longer clear how an observer can distinguish between a genuine classical object and a fluctuation in the quantum radiation. (This view has previously been advocated by Susskind.) Indeed, all of standard cosmology may be a rare fluctuation in a long-lived de Sitter space [1].

Most of high energy physics is based on the S-matrix, with the implicit assumptions that an observer of infinite size is located at the infinity of an asymptotically flat space-time — the observer is "outside looking in". This point of view will have to be transcended in order to describe experiments in cosmology, where the observer is always of finite size, and is "inside looking out". Indeed, a Minkowski infinity typically does not exist in cosmology; but even if it did, real observers would not live there. On the other hand, the approximation of an observer as a point in the space-time bulk is also unsatisfactory, because an experiment involves the collection and analysis of information. According to the holographic principle, a non-vanishing amount of information can be obtained only by an observer of non-zero size. The maximal information involved in an experiment is related not to the size of the universe, but to the size of the experiment. One may be motivated by these considerations to abandon the distinction between observer and experiment, and also to claim that a general experiment *is* a causal diamond. The bottom cone is best thought of as arising from the limitation of preparing the apparatus in a causal way; the future cone reflects the limitation of analysing the data causally.

The correspondence between finite N and positive Λ would impose a surprisingly strong restriction on the fundamental theory, if indeed we live in a universe with positive (and true) vacuum energy. We believe that its implications deserve to be further explored.

Acknowledgments

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A. de Sitter space

This appendix summarizes a number of properties of de Sitter space that are used in the text. An excellent discussion of the classical geometry is found in ref. [31]. The semi-classical properties are laid out in ref. [21].

de Sitter space is the maximally symmetric solution of the vacuum Einstein equations with a positive cosmological constant, Λ. It is positively curved with characteristic length

$$r_0 = \sqrt{\frac{3}{\Lambda}}\,. \tag{A.1}$$

Globally, de Sitter space can be written as a closed FRW universe:

$$ds^2 = -dT^2 + r_0^2 \left(\cosh \frac{T}{r_0}\right)^2 d\Omega_3^2\,. \tag{A.2}$$

The spacelike slices are three-spheres. The space-time can be visualized as a hyperboloid [31] (figure 10). The smallest S^3 is at the throat of the hyperboloid, at $T = 0$. For $T > 0$, the three-spheres expand exponentially without bound. The time evolution is symmetric about $T = 0$, so three-spheres in the past are arbitrarily large and contracting.

The Penrose diagram of de Sitter space is a square (figure 10). The spatial three-spheres are horizontal lines. As usual, every point represents a two-sphere, except the points on the left and right edge of the square, which represent the poles of the three-sphere. The top and bottom edge are the future and past infinity, where all spheres become arbitrarily large.

In de Sitter space, an observer is surrounded by a cosmological horizon at $r = r_0$. This is best seen in the static coordinate system:

$$ds^2 = -V(r)\,dt^2 + \frac{1}{V(r)}dr^2 + + r^2 d\Omega_2^2\,, \tag{A.3}$$

where

$$V(r) = 1 - \frac{r^2}{r_0^2}\,. \tag{A.4}$$

This system covers only part of the space-time, namely the interior of a cavity bounded by $r = r_0$. By the arguments given in section 2, this is precisely the operationally meaningful portion of de Sitter space, because it is the largest causal diamond possible. It corresponds to a quarter of the Penrose diagram (e.g. for an observer at the left pole, the “left triangle” shown in figure 11).

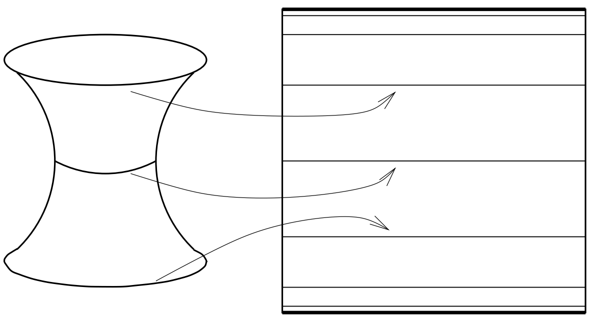

Figure 10: de Sitter space as a hyperboloid. Right: Penrosediagram. Horizontal lines represent three-spheres.

The upper and lower triangles contain exponentially large regions that cannot be observed. The spheres with $r = r_0$, the past and future event horizons, are the entire diagonals of the square. However, the spheres with $r = r_0 - \epsilon$ (the stretched horizon [26]) lie within the left (or right) triangle and represent the cosmological horizon of an observer at the corresponding pole.

An object held at a fixed distance from the observer is redshifted; the red-shift diverges near the horizon. If released, the object will accelerate towards the horizon. If it crosses the horizon, it cannot be retrieved. Thus, the cosmological horizon acts like a black hole “surrounding” the observer. Note that the symmetry of the space-time implies that the location of the cosmological horizon is observer-dependent.

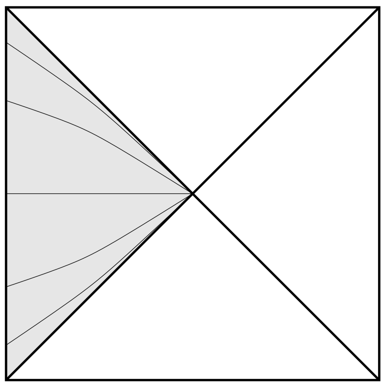

Figure 11: Past and future event horizon (diagonal lines). The static slicing covers the interior of the cosmological horizon (shaded).

The black hole analogy carries over to the semi-classical level [21]. Because matter entropy can be lost when it crosses, the cosmological horizon must be assigned a Bekenstein-Hawking entropy equal to a quarter of its area, in order for the generalized second law of thermodynamics [18, 19, 20] to remain valid:

$$S_0 = \frac{A_0}{4}\,, \tag{A.5}$$

where

$$A_0 = 4\pi r_0^2 = \frac{12\pi}{\Lambda}\,. \tag{A.6}$$

The horizon emits Hawking radiation with temperature $(2\pi r_0)^{-1}$.

de Sitter space can also be written as a flat expanding FRW universe:

$$ds^2 = -d\tau^2 + \exp\left(\frac{2\tau}{r_0}\right)\left(dx^2 + dy^2 + dz^2\right). \tag{A.7}$$

This metric covers half of the Penrose diagram (figure 12). If matter is present, it gives rise to a singularity on a space-like slice at finite time τ_0, which one can take to be 0. One thus obtains a space-time which starts with a big bang and becomes asymptotically de Sitter in the future. Its Penrose diagram is given by a portion of the flat slicing, between some finite τ and asymptotic infinity.

The remaining half of de Sitter space is covered by the contracting flat FRW universe obtained by time-reversal of eq. (A.7). By analogy with the previous paragraph, the introduction of matter leads to a flat FRW universe that is asymptotically de Sitter in the past and collapses in a big crunch, with a time-reversed Penrose diagram. (These space-times are used in section 2 to illustrate the restriction to causal diamonds.)

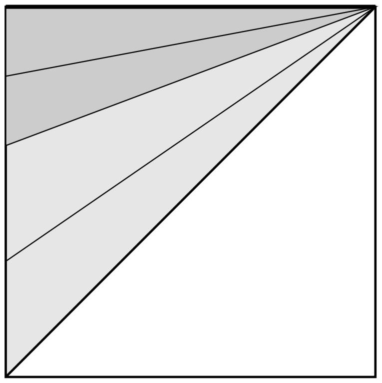

Figure 12: The flat slicing covers half of de Sitter space. The dark shaded region is the Penrose diagram of a flat big bang-de Sitter cosmology (figure 1).

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